

# Superheated Laminar Film Condensation on a Nonisothermal Horizontal Tube

Sheng-An Yang\*

National Kaohsiung Institute of Technology, Kaohsiung 807, Taiwan, Republic of China

A theoretical model is developed for the study of combined forced and natural convection film condensation from downward-flowing vapors onto a horizontal tube with variable wall temperature, including effects of superheated vapor, the pressure gradient, and vapor shear stress. The characteristics of condensate film flow and its corresponding critical angle  $\phi_c$ , where the film separates from the wall, have been investigated. The mean heat transfer result shows that, as the wall temperature variation amplitude  $A$  increases, the value of  $Nu Re^{-1/2}$  with the inclusion of the pressure gradient effect goes down appreciably; however, the value of  $Nu Re^{-1/2}$  ignoring the pressure gradient effect will remain almost uniform. Furthermore, as the vapor superheating parameter  $Sp$  increases, the mean condensing heat transfer coefficient is significantly increased about 3.4 to 18.5%, depending on  $A$ .

## Nomenclature

$A$  = wall temperature variation amplitude  
 $C_p$  = specific heat of condensate at constant pressure  
 $D$  = diameter of circular tube  
 $F$  = dimensionless parameter,  $(Ra/Ku)/Re^2$   
 $g$  = acceleration caused by gravity  
 $h$  = condensing heat transfer coefficient at angle  $\phi$   
 $\bar{h}$  = mean value of condensing heat transfer coefficient  
 $h_{fg}$  = latent heat of condensate  
 $Ku$  = Kutateladze number,  $C_p \Delta T / h_{fg}$   
 $k$  = thermal conductivity of condensate  
 $m''$  = condensate mass flux per unit area  
 $Nu$  = local Nusselt number,  $hD/k$   
 $\bar{Nu}$  = mean Nusselt number,  $\bar{h}D/k$   
 $P$  = dimensionless pressure gradient parameter,  $(\rho_v/\rho)Pr/Ku$   
 $Pe_v$  = Peclet number,  $DU_\infty/\alpha_v$   
 $Pr$  = Prandtl number of condensate,  $C_p\mu/k$   
 $p$  = static pressure of condensate  
 $q$  = local heat flux  
 $Ra$  = Rayleigh number,  $\rho(\rho - \rho_v)gPrD^3/\mu^2$   
 $Re$  = two-phase mean Reynolds number,  $\rho DU_\infty/\mu$   
 $r$  = radius of tube  
 $Sp$  = superheat parameter of vapor,  $(k_v \Delta T_v / k \Delta T) \sqrt{Pe_v/Re}$   
 $T_{sat}$  = saturation temperature of vapor  
 $T_w$  = wall temperature  
 $T_\infty$  = temperature of superheated vapor at the mainstream  
 $U_\infty$  = vapor velocity of the main freestream  
 $u_e$  = tangential vapor velocity at the edge of the boundary layer  
 $x$  = coordinate measuring distance along circumference from top of tube  
 $y$  = coordinate normal to the tube surface  
 $\alpha_v$  = thermal diffusivity of vapor  
 $\Delta T$  = temperature difference between the interface of vapor and wall,  $T_{sat} - T_w$   
 $\Delta T_v$  = temperature difference between mainstream and interface of vapor,  $T_\infty - T_{sat}$   
 $\delta$  = local thickness of condensate film  
 $\delta^*$  = dimensionless thickness of condensate film  
 $\mu$  = absolute viscosity of condensate

$\rho$  = density of condensate  
 $\rho_v$  = density of vapor  
 $\phi$  = angle measured from top of tube

## Subscripts

$c$  = critical condition  
 $sat$  = saturation  
 $v$  = vapor  
 $w$  = tube wall  
 $\infty$  = mainstream

## Superscripts

$*$  = dimensionless  
 $-$  = averaged

## I. Introduction

THE problem of laminar film condensation on horizontal cylinders submerged in a forced flow is not only of theoretical interest, but also of great importance for some technological processes, such as in the design of the condensers for powerplants and phenomenon that usually occurs in numerous engineering applications. The purpose here is to develop a simple model to predict the mean condensing heat transfer of superheated vapor on a horizontal tube with variable wall temperature and a cross forced flow.

Laminar film condensation from a saturated vapor onto an isothermal circular tube in a crossflow has received considerable attention. The earlier analytical investigation with vapor velocity effects were considered as an extension of Nusselt's analysis<sup>1</sup> to include the interfacial shear boundary condition at the edge of the condensate film. Shekrladze and Gomela<sup>2</sup> realized that the primary contribution to the surface shear was because of the change in momentum across the interface, and they employed a concept that the condensation process, in effect, is similar to suction applied around the tube on which condensation takes place and, hence, vapor boundary-layer separation can be assumed to be absent. Denny and Mills<sup>3</sup> applied the asymptotic shear model of Shekrladze and Gomela<sup>2</sup> to condensation on a circular cylinder, ignoring the pressure gradient effect, and obtained results very close to the boundary-layer solutions (within 1 or 2%). Fujii et al.<sup>4</sup> basically applied a two-phase boundary-layer flow model with the condensate film remaining intact for the entire surface, and assumed a quadratic velocity profile in the vapor boundary to investigate filmwise condensation of vapor flowing cross to a circular tube; however, they still ignored the pressure gradient.

Received Jan. 3, 1997; revision received April 18, 1997; accepted for publication April 21, 1997. Copyright © 1997 by the American Institute of Aeronautics and Astronautics, Inc. All rights reserved.

\*Associate Professor, Department of Mold and Die Engineering.

Rose<sup>5</sup> modified Shekrladze and Gomelaui's<sup>2</sup> model, and took into account the pressure gradient effect upon the same topic by using potential flow theory. Rose<sup>5</sup> proposed an empirical expression for a mean Nusselt number and found that, with inclusion of the pressure gradient, this led to a small decrease in the mean heat transfer coefficient. Afterward, Jacobi and Goldschmidt<sup>6</sup> extended both Shekrladze and Gomelaui<sup>2</sup> and Rose's<sup>5</sup> model to predict the mean heat transfer in a condensing crossflow over an infinite cylinder, by further accounting for the Marangoni effect.

All of the previously mentioned works relate to forced flowing condensation of an isothermal circular tube. Fujii et al.<sup>4</sup> showed experimentally that the peripheral distribution of wall temperature is affected by both oncoming velocity and wall heat flux. Michael et al.<sup>7</sup> also found that the difference between the vapor-saturated and local wall nonuniform temperatures is usually varying with the  $1 - A \cos \phi$  profile for the forced convection film condensation problem; whereas at high vapor velocities, the measured wall temperature profile is more uniform than predicted. Memory et al.<sup>8</sup> investigated the forced convection film condensation with a variable wall temperature in which the pressure gradient is absent. However, Memory's<sup>8</sup> higher mean condensing heat transfer coefficients seem to be in contradiction with Honda and Fujii's<sup>9</sup> lower coefficients through the conjugate approach. The pressure study with a further inclusion of a pressure gradient effect can modify Memory et al.'s<sup>8</sup> work, and can try to explain the significant discrepancy between them.

Although the effect of superheated vapor on free convection condensation on a vertical has been developed,<sup>10,11</sup> very few works regarding forced convection film condensation have been performed, especially for a circular tube. The main reason for this lies in the tedious calculation involved in solving the coupled two-phase boundary-layer equations. The present work is aimed at developing a one-phase boundary-layer model by adopting a potential flow theory and Sideman's<sup>12</sup> model for mixed convection condensation, with further inclusion of the effects of pressure gradient and vapor superheat without losing its accuracy. The present result is useful in its simplicity and easy to apply. Such a model has not previously appeared in the literature regarding forced convection film condensation on a horizontal tube.

## II. Analysis

Consider a horizontal circular tube immersed in a downward-flowing, pure vapor, which is at its superheated temperature  $T_\infty (> T_{\text{sat}})$ , and moves at  $U_\infty$ .  $T_w$  may be nonuniform and below  $T_{\text{sat}}$ . Thus, condensation occurs on the wall, and a continuous film of liquid condensate runs downward over the tube under the simultaneous effects of gravity, pressure gradient forces, and the interfacial vapor shear forces. Further, the liquid-vapor interface is assumed to be smooth and ripple-free up to separation.

The physical model under consideration is shown in Fig. 1. With the assumptions of Bromley's,<sup>13</sup> Rohsenow's,<sup>14</sup> and Sparrow and Gregg's,<sup>15</sup> works, this leads to the governing equations for the momentum and energy balance in the film:

$$m'' = \rho \frac{d}{r d\phi} \int_0^\delta u dy \quad (1)$$

$$\mu \frac{\partial^2 u}{\partial y^2} + (\rho - \rho_v)g \sin \phi - \frac{dp}{r d\phi} = 0 \quad (2)$$

$$h'_{fg} m'' - k_v \left( \frac{\partial T_v}{\partial y} \right)_\delta = -k \frac{\partial T}{\partial y} = k \frac{\Delta T}{\delta} \quad (3)$$

where  $h'_{fg} = h_{fg} + 0.68Cp\Delta T$  is the latent heat of condensation corrected for the condensate subcooling by Rohsenow.<sup>14</sup> (When not subscripted, a property is taken as that of the liquid phase.)

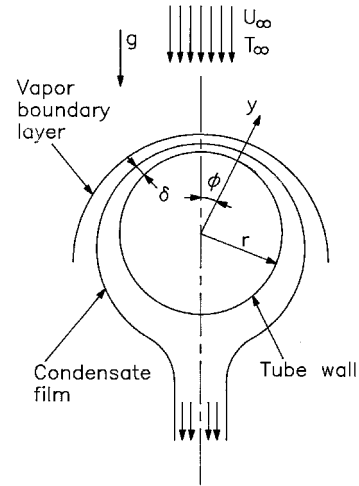


Fig. 1 Physical model and coordinate system.

Attention is given first to the energy equation [Eq. (3)]. The right-hand side is the heat conducted into the liquid. The first term on the left represents the energy liberated as latent heat; whereas the last term is the heat conducted to the interface through the vapor. The heat transferred out of the bulk vapor is found by using the solution to the energy equation of the vapor according to Sideman.<sup>12</sup> Sideman's solution<sup>12</sup> is based on potential flow, and also on the notation that the heat transfer in the vapor is confined to a thin layer near the interface. Using this yields

$$-k_v \left( \frac{\partial T_v}{\partial y} \right)_\delta = \frac{2k_v \Delta T_v \sin \phi}{\sqrt{\pi(1 - \cos \phi) D \alpha_v / U_\infty}} \quad (4)$$

Applying the Bernoulli equation to the pressure gradient term in Eq. (2) along the interface, and assuming the condensate film thickness to be neglected when compared with the radius of the tube, one may rewrite the momentum equation

$$\mu \frac{\partial^2 u}{\partial y^2} = -(\rho - \rho_v)g \sin \phi - \rho_v u_e \frac{du_e}{r d\phi} \quad (5)$$

with the following boundary conditions:

$$\frac{\partial u}{\partial y} = \frac{\tau_\delta}{\mu} \quad \text{at } y = \delta \quad (6)$$

$$u = 0 \quad \text{at } y = 0 \quad (7)$$

Consequently, the velocity profile can be obtained by integrating Eq. (5):

$$u = \frac{\tau_\delta}{\mu} - \frac{1}{\mu} \left[ (\rho - \rho_v)g \sin \phi + \rho_v u_e \frac{du_e}{r d\phi} \right] \left( \frac{y^2}{2} - y\delta \right) \quad (8)$$

To reduce a coupled two-phase boundary-layer approach into a one-phase boundary-layer approach, one may adopt Shekrladze and Gomelaui's<sup>2</sup> model for the interfacial condition, as follows:

$$\tau_\delta = m'' u_e \quad (9)$$

The previous approximation has limitations, i.e., this model only applies to high condensate rates. However, Eq. (9) is useful in its simplicity, and is also employed in Jacobi<sup>16</sup> to yield good results.

According to the potential flow theory, for a uniform flow with velocity  $U_\infty$  past a circular tube, one may derive the vapor velocity at the edge of boundary layer as

$$u_e = U_\infty 2 \sin \phi \quad (10)$$

and then obtain the pressure gradient and interface vapor shear by using Eq. (10) and assuming the film thickness of condensate  $\delta \ll r$ , as follows:

$$\rho_v u_e \frac{du_e}{r d\phi} = \rho_v U_\infty^2 4 \frac{\sin 2\phi}{D} \quad (11)$$

$$\tau_\delta = m'' U_\infty 2 \sin \phi \quad (12)$$

Next, since the wall temperature is variable, the wall temperature distribution should be specified or fitted by measured data, then one can calculate the mean wall temperature as

$$\overline{T_w} = \frac{1}{\pi} \int_0^\pi T_w(\phi) d\phi \quad (13)$$

and express the temperature difference across the film in the usual manner, which is also seen in Memory et al.<sup>8</sup>

$$\Delta T = (T_{\text{sat}} - \overline{T_w})(1 - A \cos \phi) = \overline{\Delta T}(1 - A \cos \phi) \quad (14)$$

where  $A$  is a constant ( $0 \leq A \leq 1$ ) and denotes the wall temperature variation amplitude. Inserting Eqs. (3), (4), and (10)–(12) into Eq. (8), and then integrating Eq. (1) by using this resultant Eqs. (8) and (14), along with introducing the dimensionless parameters, yields

$$2\delta^* \frac{d}{d\phi} \left[ \delta^*(1 - A \cos \phi) \sin \phi - 2Sp\delta^{*2} \frac{\sin^2 \phi}{\sqrt{\pi(1 - \cos \phi)}} + \frac{1}{3}\delta^{*3}F \sin \phi + \frac{4}{3}\delta^{*3}P \sin 2\phi \right] = 1 - A \cos \phi - 2Sp\delta^* \frac{\sin \phi}{\sqrt{\pi(1 - \cos \phi)}} \quad (15)$$

with the boundary condition

$$\frac{d\delta^*}{d\phi} = 0 \quad \text{at} \quad \phi = 0 \quad (16)$$

where

$$\delta^* = \left( \frac{\delta}{D} \right) \sqrt{Re} \quad (17)$$

$$F = (Ra/Ku)/Re^2 \quad (18)$$

$$P = (\rho_v/\rho)Pr/Ku \quad (19)$$

$$Sp = (k_v \Delta T_v / k \overline{\Delta T}) \sqrt{Pe_v / Re} \quad (20)$$

The first term inside the derivative in Eq. (15) results from the interfacial shear stress, whereas the term involving  $P$  is the effect of pressure gradient as a result of the potential flow. When both of these terms are omitted, Eq. (15) reduces to the pure free-convection film condensation, i.e., Nusselt-type condensation problem.

Applying the boundary condition, Eq. (16) into Eq. (15), one may obtain the equation for the condensate film thickness at  $\phi = 0$ ,  $\delta_0^*$ , as

$$\frac{1}{2} - \delta_0^{*2} - \frac{1}{3}(F + 8P)\delta_0^{*4} = 0 \quad (21)$$

Note that previous quadratic equation can be solved analytically, yielding  $\delta_0^{*2}$  and, hence,  $\delta_0^*$ .

Before obtaining the solution of Eq. (15), and then calculating the heat transfer rate for the tube, note that the condensate film flow may separate at the following condition:

$$\left( \frac{\partial u}{\partial y} \right)_{y=0} \leq 0 \quad (22)$$

This condition may also be obtained by Eq. (15), and result in the following relationship:

$$1 - A \cos \phi - 4\delta^* Sp \sin \phi / \sqrt{\pi(1 - \cos \phi)} + F(1 + 8P \cos \phi)\delta^{*2} = 0 \quad (23)$$

If  $\phi = \phi_c$  satisfies Eq. (23),  $\phi_c$  is called the critical angle, i.e.,

$$\frac{d\delta^*}{d\phi} \rightarrow \infty \quad \text{as} \quad \phi \rightarrow \phi_c \quad (24)$$

Although  $\delta^*$  is unknown, it may be solved by means of a fourth-order, Runge–Kutta integration that uses a step size of  $\Delta\phi = 0.05$  deg, and is then substituted into Eq. (23) by a bisection method to determine the position or value of  $\phi_c$ . The algorithm is very unstable and sensitive to the calculated  $\delta^*$  at  $\phi_c$  close to  $\phi_c$ , and so we are required to check if the condensate film thickness will abruptly become extra thick, i.e.,  $\delta^* \rightarrow \infty$ , as  $\phi \rightarrow \phi_c$ . Obviously, when  $Sp = 0$  and  $F \geq 8P$ , it satisfies  $\phi_c = \pi$ . In this case, the condensate film will separate or drip off at the bottom of the tube. Otherwise, if  $F < 8P$ , the critical angle lies in  $\pi/2 < \phi_c \leq \pi$ , and the condensate film will drip off before reaching the bottom of the tube. Because, for the latter cases, solutions will not be possible beyond  $\phi_c$ , one may ignore the contribution to heat transfer because of extra large film thickness.

The local heat flux  $q$  is given by

$$q = \frac{k\Delta T}{\delta} = \frac{k\overline{\Delta T}(1 - A \cos \phi)}{\delta} \quad (25)$$

which may be nondimensionalized to give

$$q^* = \frac{qD}{(\sqrt{Re}k\overline{\Delta T})} = \frac{(1 - A \cos \phi)}{\delta^*} \quad (26)$$

The mean heat flux for a circular tube is given by

$$\bar{q} = \frac{2}{\pi D} \int_0^\pi q \, dx = \int_0^\pi q \frac{d\phi}{\pi} \quad (27)$$

As in Nusselt's theory, the dimensionless local heat transfer coefficient can be shown to be

$$Nu = D/\delta = \sqrt{Re}/\delta^* \quad (28)$$

Next, we are interested in an expression for the overall mean heat transfer coefficient. Integrating Eq. (28) over a whole tube, but neglecting the contribution to the heat transfer beyond  $\phi_c$ , based on the whole surface area, gives

$$\overline{Nu} = \bar{q}D/k\overline{\Delta T} \quad (29)$$

$$\overline{Nu}Re^{-1/2} = \int_0^{\phi_c} \left( \frac{1}{\delta^*} \right) (1 - A \cos \phi) \frac{d\phi}{\pi} \quad (30)$$

It is to be noted, at low vapor velocity, that Eq. (30) blends with the Nusselt-type solution. Furthermore, there are two asymptotic cases that are explained as follows:

Firstly, for the case of  $F \gg 1$ , i.e., for very slow vapor flow, the gravity force is much larger than the vapor force and the pressure gradient caused by the potential flow. Hence, the problem reduces to the natural convection film condensation, i.e., Nusselt-type condensation. After omitting the first and final terms in Eq. (15), one has

$$\begin{aligned} \frac{2}{3} \delta^* \frac{d}{d\phi} \left[ \delta^{*3} F \sin \phi - 2Sp\delta^{*2} \frac{\sin^2 \phi}{\sqrt{\pi(1 - \cos \phi)}} \right] \\ = 1 - A \cos \phi - 2Sp\delta^* \frac{\sin \phi}{\sqrt{\pi(1 - \cos \phi)}} \end{aligned} \quad (31)$$

For saturated vapors,  $Sp = 0$ , by separation of variables, one may obtain

$$\delta^* = F^{-1/4} \sin^{-1/3} \phi \left[ 2 \int_0^\phi (1 - A \cos \phi) \sin^{1/3} \phi d\phi \right]^{1/4} \quad (32)$$

Thus, one can obtain the local Nusselt number from Eq. (28). Next, one may also obtain the overall mean Nusselt number from Eq. (30) as follows:

$$\begin{aligned} \overline{Nu} \left( \frac{Ku}{Ra} \right)^{1/4} = \int_0^{\phi_c} \left\{ (1 - A \cos \phi) \sin^{1/3} \phi d\phi \right. \\ \left. \times \left[ 2 \int_0^\phi (1 - A \cos \phi) \sin^{1/3} \phi d\phi \right]^{-1/4} \right\} \frac{d\phi}{\pi} \end{aligned} \quad (33)$$

It is to be noted that, for the special case of  $A = 0$  and  $Sp = 0$ ,  $\overline{Nu}(Ku/Ra)^{1/4} = 0.728$ .

Secondly, for the  $F \ll 1$  case, i.e., for the other asymptotic cases—forced convection dominated film condensation, their critical angles  $\phi_c$  are less than  $\pi$  always when  $F < 8P$ . For pure forced convection film condensation, one may put  $F = 0$  in Eq. (15) and then obtain the mean heat transfer coefficients from calculations using Eqs. (15)–(30). For the special case of  $Sp = 0$  and  $P = 0$ , the present work will reduce to be the same as Memory et al.'s<sup>8</sup> results.

### III. Results and Discussion

The present results cover the cases of combined free and forced convection film condensation of superheated vapor flowing downward onto a horizontal tube. Here, numerical results for mixed convection film condensation of superheated vapor flowing downward onto a horizontal tube are presented in two parts. In the first part, numerical results of local condensate film thickness and its corresponding critical angles are obtained and discussed for a wide range of  $F$ ,  $P$ ,  $A$ , and  $Sp$  beyond the practical range. In practice,  $P$  is usually less than 1.0 for steam and less than 4.0 for refrigerants.  $Sp$  is taken to vary from 0 to 1.0, and so is  $A$ . Then, the second part will indicate and discuss the performance of heat transfer rates or Nusselt number for the same range of  $F$ ,  $P$ ,  $A$ , and  $Sp$  as the first part.

#### A. Characteristics of Flow Dynamics $\delta^*$ and $\phi_c$

##### 1. Distribution of Condensate Film Thickness

Firstly, for  $A = 0$  cases, i.e., for isothermal wall cases, the results of numerical solutions from Eq. (15) are shown in Fig. 2a. For larger values of  $Sp$ , the superheat effect makes the condensate film thickness thinner for all values of  $F$ . The reason for this is that some of the heat conducted through the condensate film layer is taken from the superheated vapor, rather than condensing the saturated vapor. Hence, the amount

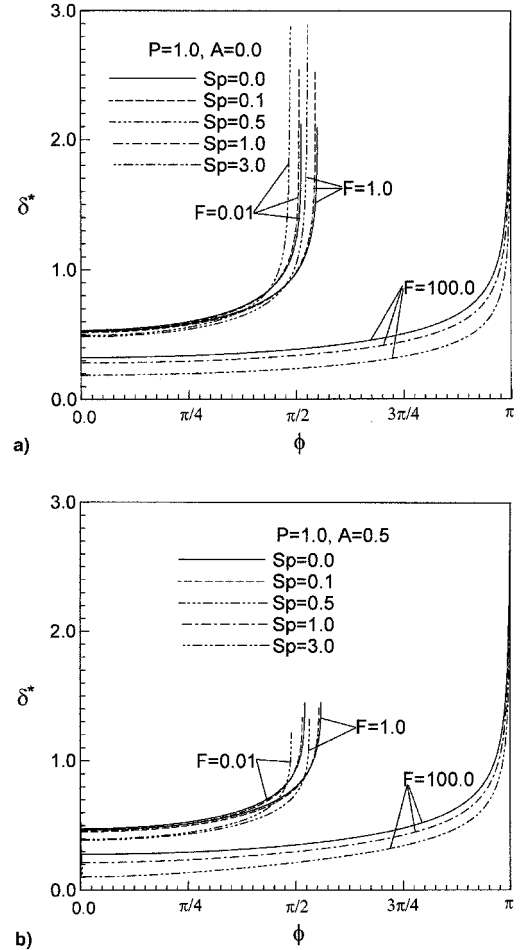


Fig. 2 Dependence of dimensionless local film thickness on angle for a) isothermal and b) variable temperature wall cases.

of condensate mass will be reduced so that the condensate film becomes thinner. For the  $F = 100$  case, i.e., free convection dominated film condensation, the condensate film remains intact throughout the surface, without separation occurring in advance. But for both  $F = 1.0$  (mixed convection) and  $F = 0.01$  (forced convection) cases, the condensate film will separate from the tube before reaching the bottom of the tube (at  $\phi_c \approx \pi/2$ ). Secondly, for  $A = 0.5$ , i.e., variable wall temperature cases, the profiles of  $\delta^*$  are shown in Fig. 2b and are similar to Fig. 2a; for all  $A = 0.5$  cases, the values of  $\delta^*$  are smaller than those for the corresponding cases of  $A = 0$ .

##### 2. Phenomena of Film Flow Separation: Values of Critical Angles

Figures 3a and 3b show the dependence of critical or separation angles of the condensate film on parameter  $F$ , for both isothermal ( $A = 0$ ) and variable temperature walls ( $A = 0.5$ ), respectively. In general, with the inclusion of the pressure gradient effect; e.g.,  $P = 1.0$  and  $3.0$ ,  $\phi_c$  increases slowly with increasing  $F$ , for smaller  $F$  or forced convection dominated film condensation. As  $F$  increases to near  $8P$ ,  $\phi_c$  is quickly increasing until  $\phi_c = \pi$ . There exist the transition zones (near  $F < 8P$ ) for isothermal and variable temperature wall cases from  $F = 1$  to  $F = 10$  around. When  $F < 1.0$ , the film condensation will be forced convection dominated; the  $\phi_c$  are about 1.6 rad for  $P = 3$  and 1.7 rad for  $P = 1$ , respectively. Because the rate of condensate mass flow is decreased with the degree of vapor superheating,  $\phi_c$  decreases slightly with  $Sp$ . For  $F > 10$ , the film is free convection dominated, and  $\phi_c = \pi$  for both  $P = 1.0$  and  $3.0$ . Further, at  $F = 10$ , the film for  $P = 3.0$  separates earlier or at smaller  $\phi_c$  than when  $P = 1.0$ .

Figure 4 shows the effect of the pressure gradient on the critical angle, i.e., the dependence of  $\phi_c$  on parameter  $P$

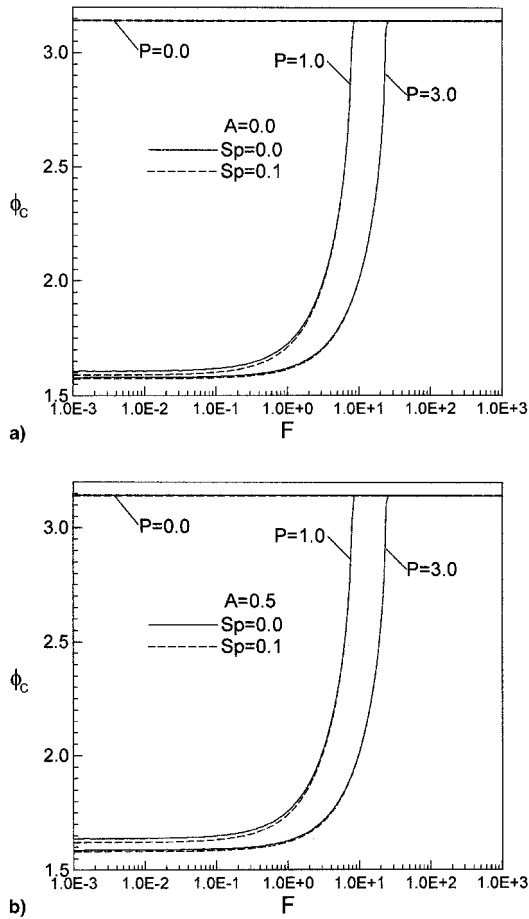


Fig. 3 Dependence of critical angle on  $F$  for a) isothermal and b) variable temperature wall cases.

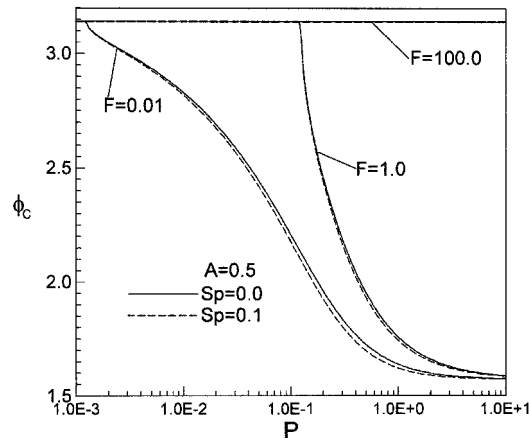


Fig. 4 Dependence of critical angle on  $P$  for mixed-convection film condensation.

for different values of  $F$ . It is seen that for  $F = 0.01$ ,  $\phi_c$  decreases continuously with  $P$ ; i.e.,  $\phi_c$  is usually less than  $\pi$ . The greater  $F$  increases, the wider the range of  $P$  that satisfies  $\phi_c = \pi$ . Additionally, at higher  $F$ ,  $\phi_c$  is almost unaffected by varying  $Sp$ . According to Memory and Rose's study,<sup>17</sup> in the case of vapor flow past a circular tube, separation occurs from  $\phi_s = 1.76$  to  $2.923$  rad, for the forced flow dominated condensation case. Compared to the present calculated values of  $\phi_c$ , the separation point  $\phi_s$  of vapor boundary layer occurs more downstream than the liquid film layer  $\phi_c$  does. Hence, the validity of the present numerical results can be guaranteed.

## B. Performance of Heat Transfer: Dimensionless Mean Heat Transfer $\overline{Nu} Re^{-1/2}$

### 1. Effect of $F$ on Mean Heat Transfer

The dimensionless mean heat transfer coefficients or mean Nusselt numbers are obtained numerically from Eq. (30) by a step size of  $\Delta\phi = 0.05$  deg. Note that the present integration with a step size twice as large as  $\Delta\phi = 0.05$  deg, produced a change in the predicted mean Nusselt number of less than 0.1%. In general, the convective heat transfer coefficient because of a single-phase condensate (for  $\phi > \phi_c$ ), is much less than the condensing heat transfer coefficient caused by phase change (for  $\phi < \phi_c$ ). Hence, ignoring the contribution to the mean heat transfer for  $\phi > \phi_c$  will not cause a significant error.

Firstly, Figs. 5a and 5b indicate the dependence of the dimensionless mean heat transfer coefficient on the effect of vapor flow velocity for isothermal and variable wall temperature, respectively. For  $P = 0$  and  $Sp = 0$  (no pressure gradient effect included and for saturated vapor), the mean Nusselt result is just the same as Memory et al.'s<sup>8</sup> (Fig. 5b). In addition, for the isothermal wall case ( $A = 0$ ), the present result reduces to Shekrladze and Gomelauroi's model.<sup>2</sup> However, Memory et al.<sup>8</sup> used the Shekrladze and Gomelauroi's shear stress approximation,<sup>2</sup> without considering pressure gradient effects, and so they overestimated the mean vapor-side heat transfer coefficient. Hence, the present results with further accounting for the pressure gradient (including  $P = 1.0$  and  $3.0$ ) make the mean heat transfer coefficients lower and reasonable, above all for lower  $F$ . It is also seen that there exists a transition zone near the Nusselt equation between  $F = 1 - 10$ .

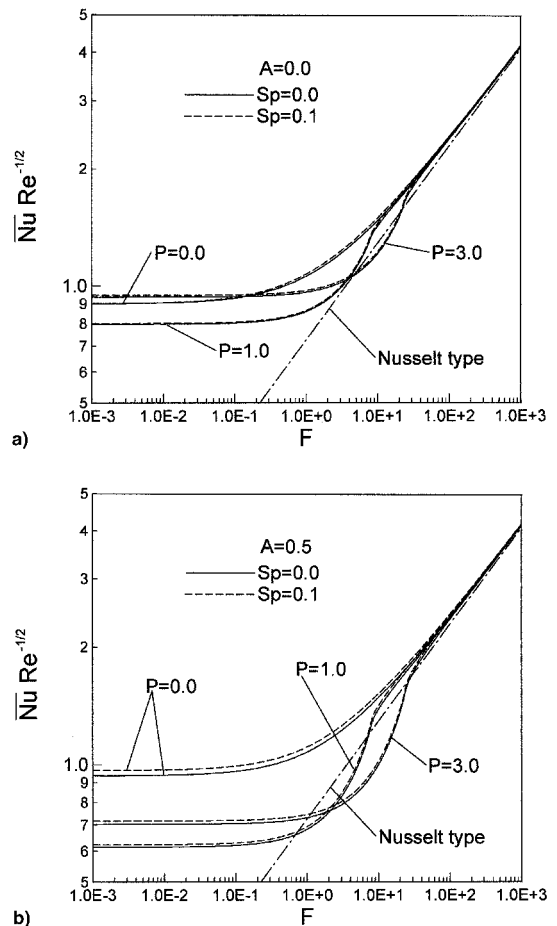


Fig. 5 Dependence of mean Nusselt number on  $F$  for a) isothermal and b) variable temperature wall cases.

## 2. Effect of $A$

Figure 6 shows that the mean Nusselt number increases insignificantly with  $A$  for  $F = 100$ . For smaller values of  $F$ , i.e.,  $F \leq 1.0$ , the mean Nusselt numbers decrease more appreciably with increasing  $A$  for forced convection dominated ( $F < 1.0$ ) condensation. Such a decrease in the mean Nusselt number is because its mean heat flux is multiplied by the factor  $1 - A \cos \phi$ . Note that the result is in contradiction with that obtained from Memory et al.,<sup>8</sup> in which the pressure gradient is absent. This is the reason that if one ignores the pressure gradient, i.e., letting  $P = 0$  for low  $F$  cases, the mean Nusselt number will be overpredicted.

## 3. Effect of $Sp$

Finally, the effect of superheated vapor on the mean heat transfer coefficient is illustrated in Fig. 7 as follows. For  $F = 100$  (free convection dominated), the dimensionless mean heat transfer coefficients or mean Nusselt numbers increase by 14.8 and 18.5%, as  $Sp$  increases from 0 to 1.0 for  $A = 0$  and 0.5, respectively. For mixed convection film condensation ( $F = 1.0$ ) and forced convection dominated film condensation ( $F = 0.01$ ), its practical ranges of superheat parameter are more limited ( $Sp < 0.1$ ), and smaller because of larger film Reynolds numbers for the forced flow of vapor. Therefore, one may check the percentage of change in mean Nusselt numbers between  $Sp = 0$  (saturated case) and 0.5 (superheated case) as follows:

1)  $F = 1.0$ , its percentage is 4.37% for the isothermal wall case ( $A = 0$ ). When  $A = 0.5$ , its percentage is 11.26%.

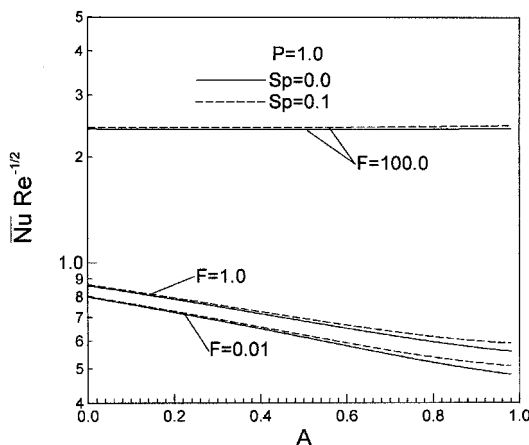


Fig. 6 Dependence of mean Nusselt number on  $A$  for mixed convection film condensation.

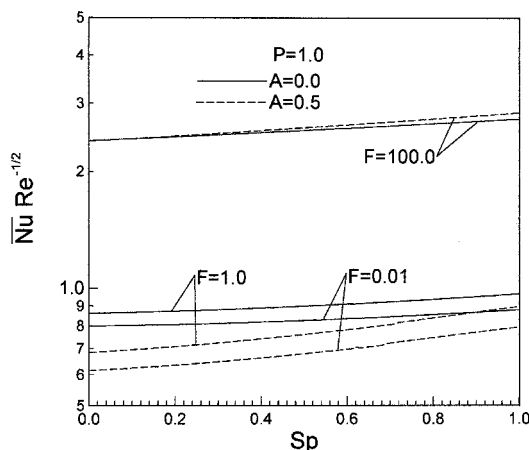


Fig. 7 Dependence of mean Nusselt number on  $Sp$  for mixed convection film condensation.

2)  $F = 0.01$ , its percentage for  $A = 0$  is 3.4%, while when  $A = 0.5$ , its percentage becomes 10.29%.

Such an increase in mean heat Nusselt numbers may be because its condensate film thickness is reduced with an increase in the degree of vapor superheating.

## IV. Concluding Remarks

1) An increase in vapor superheating leads to an enhancement of the condensation heat transfer coefficient.

2) For  $Sp = 0$ , or saturated vapor case, the present model with further inclusion of the pressure gradient effect has explained the discrepancy between Memory et al.<sup>8</sup> (ignoring the pressure gradient term) and Honda and Fujii's<sup>9</sup> work (using actual shear stress); the mean heat transfer result appears to be adequate and can be applied to the combined free convection and forced convection film condensation.

3) The dependence of the critical angle of condensate film  $\phi_c$  on the pressure gradient has been investigated by the present approach. Note that for  $P = 0$ , the present result yields  $\phi_c = \pi$ , which is also found in Memory et al.'s model.<sup>8</sup>

4) When  $P = 0$ , the mean heat transfer coefficient increases insignificantly with  $A$ . The mean heat transfer coefficient decreases appreciably with  $A$  as  $P$  is increased from 0.

5) The mean heat transfer coefficient is nearly unaffected by the pressure gradient term for gravity-dominated condensation. For lower  $F$  (higher vapor velocity), the mean heat transfer coefficient decreases significantly with increasing pressure gradient effect.

6) Because of the neglected waviness in the condensate film layer, the model should be cautiously applied when  $Re$  approaches values where interfacial waves exist.

## References

- Nusselt, W., "Die Oberflächen Kondensation des Wasserdampfes," *Zeitschrift des Vereines Deutscher Ingenieure*, Vol. 60, No. 4, 1916, pp. 541–546; 569–575.
- Shekriladze, I. G., and Gomelauro, V. I., "Theoretical Study of Laminar Film Condensation of Flowing Vapour," *International Journal of Heat and Mass Transfer*, Vol. 9, 1966, pp. 581–591.
- Denny, V. E., and Mills, A. F., "Laminar Film Condensation of a Flowing Vapor on a Horizontal Cylinder at Normal Gravity," *Journal of Heat Transfer*, Vol. 91c, 1969, pp. 495–501.
- Fujii, T., Uehara, H., and Kruata, C., "Laminar Film Condensation of Flowing Vapour on a Horizontal Cylinder," *International Journal of Heat and Mass Transfer*, Vol. 15, 1972, pp. 235–246.
- Rose, J. W., "Effect of Pressure Gradient in Forced Convection Film Condensation on a Horizontal Tube," *International Journal of Heat and Mass Transfer*, Vol. 27, No. 1, 1984, pp. 39–47.
- Jacobi, A. G., and Goldschmidt, V. W., "The Effect of Surface Tension Variation on Filmwise Condensation and Heat Transfer on a Cylinder in Cross-Flow," *International Journal of Heat and Mass Transfer*, Vol. 32, No. 8, 1989, pp. 1483–1490.
- Michael, A. G., Rose, J. W., and Daniels, L. C., "Forced Convection Condensation on a Horizontal Tube—Experiment with Vertical Downflow of Steam," *Journal of Heat Transfer*, Vol. 111, Aug. 1989, pp. 792–797.
- Memory, S. B., Lee, W. C., and Rose, J. W., "Forced Convection Film Condensation on a Horizontal Tube—Effect of Surface Temperature Variation," *International Journal of Heat and Mass Transfer*, Vol. 36, No. 6, 1993, pp. 1671–1676.
- Honda, H., and Fujii, T., "Condensation of Flowing Vapor on a Horizontal Tube—Numerical Analysis as a Conjugate Heat Transfer Problem," *Journal of Heat Transfer*, Vol. 106, 1984, pp. 841–848.
- Sparrow, E. M., and Eckert, E. R. G., "Effects of Superheated Vapor and Noncondensable Gases on Laminar Film Condensation," *AIChE Journal*, Vol. 7, No. 3, 1961, pp. 473–477.
- Spencer, D. J., and Ibele, W. E., "Laminar Film Condensation of a Saturated and Superheated Vapor," *3rd International Heat Transfer Conference* (Chicago, IL), 1966, pp. 337–342.
- Sideman, S., "The Equivalence of the Penetration Theory and Potential Flow Theories," *Industrial and Engineering Chemistry*, Vol. 58, No. 2, 1966, pp. 54–58.

<sup>13</sup>Bromley, L. A., "Effect of Heat Capacity of Condensate," *Industrial and Engineering Chemistry*, Vol. 44, 1952, pp. 2966–2969.

<sup>14</sup>Rohsenow, W. M., "Heat Transfer and Temperature Distribution in Laminar Film Condensation," *Transactions of the American Society of Mechanical Engineers*, Vol. 78, 1956, pp. 1645–1648.

<sup>15</sup>Sparrow, E. M., and Gregg, J. L., "A Boundary Layer Treatment of Laminar-Film Condensation," *Transactions of the American*

*Society of Mechanical Engineers*, Vol. 81, 1959, pp. 13–17.

<sup>16</sup>Jacobi, A. M., "Filmwise Condensation from a Flowing Vapor onto Isothermal, Axisymmetric Bodies," *Journal of Thermophysics and Heat Transfer*, Vol. 6, No. 2, 1992, pp. 321–325.

<sup>17</sup>Memory, S. B., and Rose, J. W., "Forced Convection Film Condensation on a Horizontal Tube-Influence of Boundary-Layer Separation," *Journal of Heat Transfer*, Vol. 117, May 1995, pp. 529–533.